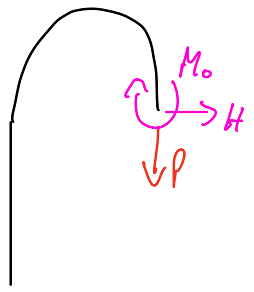
## 10.2




$U = U(M_0, H, P)$

$\delta_v = \dfrac{\partial U}{\partial P}$ je cherche $M_f(x)$

$\delta_H = \left.\dfrac{\partial U}{\partial H}\right|_{H=0 \text{ et } M_0=0}$

$\alpha = \left.\dfrac{\partial U}{\partial M_0}\right|_{M_0=0 \text{ et } H=0}$

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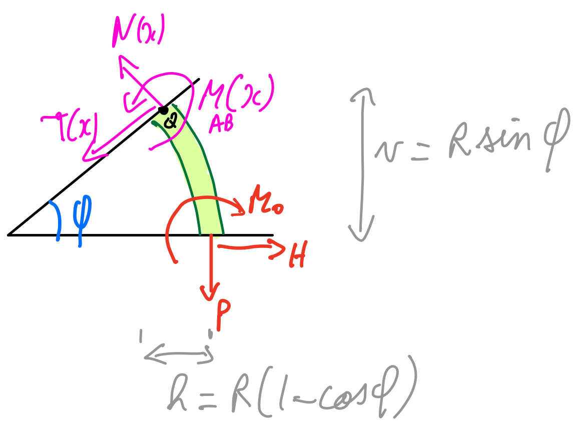
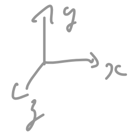
PARTIE AB COURBE



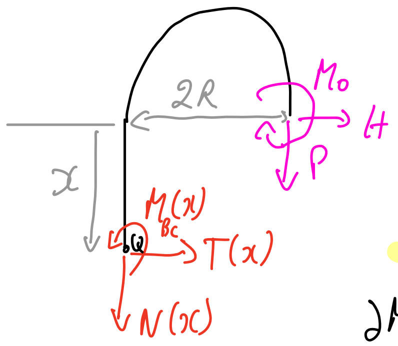

$\sum M_{z_Q} = 0 \qquad -M_0 + M_{AB}(x) - P \cdot h + H \cdot v = 0$

$$M_{AB}(x) = PR(1-\cos\varphi) - HR\sin\varphi + M_0$$

$$\left.M_{AB}(x)\right|_{H=0,\, M_0=0} = PR(1-\cos\varphi)$$

$\dfrac{\partial M_{AB}}{\partial H} = -R\sin\varphi \qquad \dfrac{\partial M_{AB}}{\partial P} = R(1-\cos\varphi) \qquad \partial M_{AB}/\partial M_0 = 1$

## PARTIE DROITE BC




$$\sum M_{z_Q} = 0$$

$$-M_0 + M_{BC}(x) - 2RP - Hx = 0$$

$$M_{BC}(x) = Hx + 2PR + M_0$$

$$\partial M_{BC}/\partial P = 2R \qquad \partial M_{BC}/\partial H = x \qquad \frac{\partial M_{BC}}{\partial M_0} = 1$$

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## DEPLACEMENT HORIZONTAL

$$\delta_H = \frac{\partial U}{\partial H}\Big|_{\substack{H=0\\ M_0=0}} = \int_A^B \frac{\overset{H=0,\,M_0=0}{M_{AB}}}{EI}\,\frac{\partial M_{AB}}{\partial H}\,\overset{R\,d\varphi}{dx}$$

$$+ \int_C^B \frac{\overset{H=0\quad M_0=0}{M_{BC}}}{EI}\,\frac{\partial M_{BC}}{\partial H}\,dx$$

$$\delta_H = \frac{1}{EI}\int_0^\pi \left[PR(1-\cos\varphi)(-R\sin\varphi)\right]R\,d\varphi + \int_0^h \frac{2PR}{EI}\,x\,dx$$

$$= -\frac{PR^3}{EI}\underbrace{\int_0^\pi (1-\cos\varphi)\sin\varphi\,d\varphi}_{2} + \frac{2PR}{EI}\underbrace{\int_0^h x\,dx}_{h^2/2}$$

$$\boxed{\delta_H = \frac{PR}{EI}\left(h^2 - 2R^2\right)}$$

vers droite car sens de H

$$\left(\int_0^\pi \sin\varphi = -\cos\varphi\Big|_0^\pi = 2\right)$$

$$\int \cos\varphi\sin\varphi = \int \frac{\sin 2\varphi}{2} = \cos 2\varphi\Big|_0^\pi = 0$$

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## DEFORMEE VERTICALE

$$\delta_V = \int_A^B \frac{M_{AB}}{EI}\,\frac{\partial M_{AB}}{\partial P} + \int_B^C \frac{M_{AB}}{EI}\,\frac{\partial M_{BC}}{\partial P}$$

$$= \int_0^{\pi} \frac{PR}{EI}(1-\cos\varphi) \cdot R(1-\cos\varphi)\, R\, d\varphi$$

$$+ \int_0^{h} \frac{2PR}{EI}\, 2R\, dx$$

$$\boxed{\delta_V = \frac{PR^2}{2EI}(3\pi R + 8h)}$$

vers le bas, car sens de P

DEFORMÉE ANGULAIRE

$$\alpha = \int_A^B \frac{M_{AB}}{EI} \frac{\partial M_{AB}}{\partial M_0} dx + \int_B^C \frac{M_{BC}}{EI} \frac{\partial M_{BC}}{\partial M_0} dx$$

H=0 $M_0=0$     H=0 $M_0=0$

$$= \int_0^{\pi} \frac{PR}{EI}(1-\cos\varphi)(1)\, R\, d\varphi + \int_0^{h} \frac{2PR}{EI} dx$$

$$\alpha = \frac{PR}{EI}(2h + \pi R)$$

$>0$ sens $M_0$